Submillimeter-Wave Production by Upshifted Reflection from a Moving Ionization Front

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Abstract—**The Doppler shift and reflection coefficients are calculated for reflection of an electromagnetic (EM) wave from a moving ionization front in a stationary gas. It is suggested that this process can be used to upshift microwaves to submillimeter waves.**

Introduction

An IONIZATION WAVE can be propagated through a gas in many ways, e.g., a strong electromagnetic (EM) pulse, an ionizing shock, a propagating electron beam, a laser or electron beam sweeping across the gas, or a programmed sequence of laser pulses [1]. The velocity U of the ionization front can be relativistic, or even (in the latter cases) exceed c. Similarly, a finite time after such an ionizing pulse has passed, a recombination wave will move across the plasma. In this paper, we consider the reflection of an EM wave, incident from the neutral gas side, by such a moving ionization or recombination front. It would appear that an electron density which will render the plasma overdense to microwaves can be attained with relatively modest means. We suggest, therefore, that the Doppler shift which occurs upon reflection can be used to upshift microwaves to the submillimeter-wave range.[1]

For specificity, we assume that the ionization or recombination front is planar, and that the EM wave is normally incident. In the present calculations, the plasma is treated as cold and collisionless. The calculations are most easily performed in the frame of reference in which the ionization or recombination front is stationary [i.e., the electron density $n(z)$ is time independent], and the gas/plasma is streaming at velocity U, as illustrated in Fig. 1.

The reflection process under consideration has some similarities to and some differences from the process of reflection by a moving mirror. A relevant example of the latter is upscattering of microwaves by reflection from a relativistic electron beam [2]. In both cases, reflection is from the electron density profile, rather than from individual particles, and thus the frequency ω_r of the reflected wave is Doppler shifted relative to the incident ω_i:

$$\omega_r/\omega_i = (1 + \beta)/(1 - \beta) \tag{1}$$

where $\beta = U/c$. In both cases, the duration τ_r of the reflected pulse is similarly related to that of the incident pulse

$$\tau_r/\tau_i = (1 - \beta)/(1 + \beta). \tag{2}$$

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[1] Downshift of an IR laser beam off a moving recombination front may also prove useful.

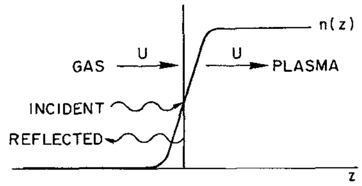

Fig. 1. The scattering problem in the frame of reference of the ionization front.

However, the plasma itself is moving in the case of the moving mirror (e.g., relativistic electron beam) but is stationary in the case considered here. This introduces a difference in the wave dispersion properties in the plasma which, in the case of an *oncoming* ionization front ($U > 0$ in Fig. 1), reduces the reflection coefficient as compared to what it would be for an oncoming mirror. Indeed it is obvious that this must be true, since the pulse reflected from a moving mirror has more energy than the incoming pulse, the extra energy being supplied by the kinetic energy of the mirror; in the case of reflection from a moving ionization front in a *stationary* gas, this energy source is not available. However, the reflection coefficient from a *receding* recombination front is shown to be exactly the same as that from a receding mirror.

Although the reflection coefficient is smaller for an oncoming ionization front than for an oncoming overdense electron beam, the *efficiency*, i.e., the reflected EM wave energy divided by the *total* energy input (incoming wave, plus beam energy or energy invested in ionization) of the two schemes may, in practical cases, be comparable. Furthermore, the ionization front scheme has advantages with respect to size and cost.

Analysis

We perform the analysis in the frame of reference in which the ionization front is stationary (Fig. 1). We make no assumptions about the shape or width of the ionization front; it may be sharp or broad, the only requirements being that $n(z) \to 0$ for $z \to -\infty$, and $n(z) \to$ constant for $z \to +\infty$. We let $\boldsymbol{J}(z,t) = -n(z)e\boldsymbol{v}(z,t)$ be the oscillating, transverse plasma current; $\boldsymbol{v}(z,t)$ is thus the oscillating component of the electron fluid (i.e., local mean) velocity, which is determined by the momentum conservation equation

$$n \frac{\partial \boldsymbol{v}}{\partial t} + nU \frac{\partial \boldsymbol{v}}{\partial z} + U\boldsymbol{v} \frac{dn}{dz} = -\frac{e}{m\gamma}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{U} \times \boldsymbol{B}\right) + \boldsymbol{R}. \quad (3)$$

For the case of a recombination front ($U < 0$), $\boldsymbol{R} = U\boldsymbol{v}(dn/dz)$ represents the loss of electron oscillating momentum upon recombination; for the case of an ionization front ($U > 0$), $\boldsymbol{R} = 0$, since electrons are born (ionized) with $\boldsymbol{v} = 0$. Using (3) and Maxwell's equations for curl $\boldsymbol{E}$ and curl $\boldsymbol{B}$, assuming time dependence $\exp(-i\omega t)$, and performing some algebraic manipulations, we find the wave equations

$$(c^2D^2 + \omega^2 - \omega_p^2)(-i\omega + UD)E = 0, \qquad U > 0 \quad (4a)$$

$$(-i\omega + UD)\omega_p^{-2}(c^2D^2 + \omega^2 - \omega_p^2)E = 0, \qquad U < 0 \quad (4b)$$

where $D \equiv d/dz$, $\omega_p^2 \equiv 4\pi n(z)e^2/m\gamma$, and $\gamma^{-2} \equiv 1 - \beta^2$. Note that ω is related to the incident frequency in the laboratory frame by $\omega = \omega_i(1 + \beta)^{1/2}/(1 - \beta)^{1/2}$, and ω_p is numerically the same as the plasma frequency in the laboratory frame.

In the gas (assumed to have the dielectric properties of the vacuum), i.e., at $z \to -\infty$, the solutions are, of course, the incident and reflected EM waves, $k = \pm\omega/c$. In the plasma, i.e., at $z \to +\infty$, where ω_p is constant, there are three solutions: (if $\omega_p^2 > \omega^2$) the evanescent and purely growing waves, $k = \pm i(\omega_p^2 - \omega^2)^{1/2}/c$, and a propagating wave $k = \omega/U$ which convects with the plasma. The first two waves have exactly the same dispersion relation as in a stationary plasma. The third wave would reduce to a time-independent magnetic field $\boldsymbol{B} = B_0\hat{y}\exp(ikz)$ in a stationary plasma, but in a moving plasma it is a genuine transverse EM wave, transporting energy at the plasma velocity U. We shall refer to it as the magnetic wave.

We now calculate the reflection coefficient by solving (4) exactly. We recall that no assumptions have been made as to whether the front is sharp or broad. We consider first the case $U < 0$. For $z \to +\infty$, where ω_p becomes constant, one of the three independent solutions of the third-order differential equations (4b) is proportional to $\exp(i\omega z/U)$, i.e., represents the magnetic wave. This solution must be excluded, since, for $U < 0$, it transports energy in from $z = +\infty$. The other two solutions satisfy

$$[d^2/dz^2 + (\omega^2 - \omega_p^2)/c^2]E = 0 \quad (5)$$

which is identical to the wave equation for the well-known problem of a stationary ionization front in a stationary plasma (i.e., a mirror). The solution to (5) in the gas at $z \to -\infty$ is thus of the form

$$E(z) = E_i\,[\exp(i\omega z/c) + \Gamma\exp(-i\omega z/c)]. \quad (6)$$

In general, the stationary reflection coefficient Γ depends on the form of $n(z)$, and the calculation of Γ is, in general, a difficult but well-known mathematical problem. However, there are two cases in which the evaluation of $|\Gamma|$ becomes simple. First, in the limit of a sharp ionization front (i.e., narrower than a wavelength),

$$\Gamma = \frac{1 - (1 - \omega_p^2/\omega^2)^{1/2}}{1 + (1 - \omega_p^2/\omega^2)^{1/2}}. \quad (7)$$

Second, $|\Gamma| = 1$ if the plasma is overdense ($\omega_p > \omega$), whatever the detailed form of $n(z)$. In these two cases, we shall produce an exact solution to our problem. In the general case, we reduce the problem of interest to the calculation of Γ, i.e., to the solution of the reflection problem at a stationary front in a stationary plasma. The most interesting case, of course, is the overdense case, in which reflection is maximized. We note that the criterion for overdense is $\omega_p > \omega$ in the front frame, i.e., $\omega_p > \omega_i(1 + \beta)^{1/2}/(1 - \beta)^{1/2}$ in the laboratory frame.

The ratio of reflected to incident power (still in the frame of the recombination front) is

$$P_r/P_i = |E_r|^2/|E_i|^2 = |\Gamma|^2. \quad (8)$$

Lorentz transforming back to the laboratory frame, where the plasma is stationary but the recombination front is receding, we have for reflected power

$$P_r^*/P_i^* = |\Gamma|^2(1 - |\beta|)^2/(1 + |\beta|)^2 \quad (9a)$$

and for total energy (integrated over the pulse)

$$\varepsilon_r^*/\varepsilon_i^* = |\Gamma|^2(1 - |\beta|)/(1 + |\beta|). \quad (9b)$$

Equations (9a) and (9b) are compatible because the reflected pulse is elongated according to (2). Equations (5)–(9) are all identical to those for a receding mirror.

We now consider the case of an oncoming ionization front $U > 0$. E is a solution of (4a) if

$$E = \int_{-\infty}^{z} dz'\,\exp[i\omega(z - z')/U]\,g(z') \quad (10)$$

and g is a solution of (5). Using the solution (6) in (10) and performing the integration, we find[2] in the gas, i.e., for $z \to -\infty$,

$$E = E_i\,[\exp(i\omega z/c) + \Gamma[(1 - \beta)/(1 + \beta)]\exp(-i\omega z/c)]. \quad (11)$$

Thus the reflected power is given by

$$P_r/P_i = |\Gamma|^2(1 - \beta)^2/(1 + \beta)^2. \quad (12)$$

Transforming back to the laboratory frame, we find that the power reflection coefficient is identical to what it would be for a stationary front

$$P_r^*/P_i^* = |\Gamma|^2 \quad (13a)$$

but the total reflected energy is less than the incident energy

$$\varepsilon_r^*/\varepsilon_i^* = |\Gamma|^2(1 - \beta)/(1 + \beta). \quad (13b)$$

[2] Equation (11) is the solution of (4a) which satisfied the boundary conditions that the magnetic wave vanishes at $z = -\infty$, and the growing wave (if the plasma is overdense) or the leftward propagating EM wave (if the plasma is underdense) vanish at $z = +\infty$.

Once again, (13a) and (13b) are compatible because the reflected pulse is shortened according to (2). Equations (13) differ from those for a moving mirror, which would be

$$P_r^*/P_i^* = |\Gamma|^2(1 + \beta)^2/(1 - \beta)^2 \tag{14a}$$

$$\varepsilon_r^*/\varepsilon_i^* = |\Gamma|^2(1 + \beta)/(1 - \beta). \tag{14b}$$

Our derivation of (7)–(9) and (11)–(13) has been quite general; however, it is of pedagogical interest to note that in the special case of a sharp ionization or recombination front [3], these equations could also be derived in a more standard way, by matching gas and plasma solutions at the front. In doing this, one must realize that for the sharp ionization front case, there is an extra (as compared to the usual problem of reflection at a stationary ionization front) solution in the plasma—the magnetic wave. Thus one more continuity condition is needed at the front, in addition to the usual conditions of continuity of E and H. This condition is that $J = 0$ at the front (since electrons are born with no oscillating velocity). From this, it follows that $\partial H/\partial z$ is also continuous.

We note from (13b) that the reflected energy is less than the incident energy, even when the usual EM wave in the plasma is cut off ($\omega < \omega_p$) and cannot carry away the excess energy. The explanation for this is as follows. For $U > 0$, and in the limit of a sharp ionization front, explicit solution of (4a) shows [3] that both the evanescent wave and the magnetic wave are excited in the plasma. The latter transmits energy (both electromagnetic energy and kinetic energy of transverse electron currents) into the plasma. In the laboratory frame, it would be more correct to say that magnetic energy and transverse electron kinetic energy are left behind in the plasma as the front moves on. In the opposite limit of a broad front, kinetic energy of transverse electron flow convects into the plasma, but the flow of different electrons is not coherent; thus the transverse current, and the amplitude of the magnetic wave, become vanishingly small. In either case, wave energy penetrates an overdense plasma, and overall conservation of energy and momentum can be demonstrated explicitly [3]. The external energy source required to ionize the gas plays no role in the wave energy balance.

For the recombination front case $U < 0$, kinetic energy of transverse electron oscillation is carried toward the neutral gas, and is ultimately released as recombination radiation. Again, it can be shown that energy and momentum are conserved [3]. The magnetic wave is never excited, since it would carry energy in from $z = +\infty$.

It is physically obvious that EM waves cannot be reflected from an oncoming ionization front moving faster than c, since the front would instantaneously overtake the wave. What happens mathematically is that, in the superluminous case, the evanescent and growing waves in the plasma both become (in the laboratory frame) propagating waves. Both of these are excited, rather than one reflected wave and one evanescent wave.

Conclusions

Microwaves reflected from an approaching ionization front will be Doppler upshifted, as given in (1). The reflection coefficient, given by (13a) and (13b), becomes small in the relativistic limit $\beta \to 1$. Nevertheless, it may be possible to produce very impressive power levels in the submillimeter regime, using recently developed high-power microwave sources. Furthermore, this technique promises to be precisely tunable, and to require a very modest investment in energy and equipment, compared to other methods of upshifting microwaves, e.g., [3].

Finally, we note that several calculations appear in the literature of reflection of EM waves from a moving discontinuity between stationary *dispersionless* dielectrics [4]. This model leads to a reflection coefficient equal to that at a physically moving dielectric surface, i.e., our (14), and thus different from our results, i.e., (12). The macroscopic model of a dispersionless dielectric interface must be examined carefully to determine whether it applies to any given physical situation; in particular, it is clearly inapplicable to the case of an ionization front. This matter will be discussed more fully in a future publication [3].

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